(Ref. 1, p. 24). We cannot share this view because of a) the aforementioned uncertainty of deductions based on cemented-burner data, b) our inability to discern an obvious maximum using uncemented burners, and c) the afterburning of fuel gas under conditions that should be lean according to this hypothesized stoichiometry. The fact that Burger failed to observe afterburning with methane (under conditions similar to those for which we observed it) is still, however, troublesome. Current investigations at our laboratory are aimed at resolving such questions of stoichiometry.

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Comment on "Large-Amplitude Transverse Instability in Rocket Motors"

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THE coupling of longitudinal and transverse modes of high-frequency combustion instability was treated theoretically by Temkin[1,2] in an effort to explain the experimental observations of Crump and Price.[3] The object of this comment is to point out that mode coupling is not unique to solid-propellant rocket combustors, but also has been observed in gas-burning rockets.

Osborn and Bonnell[4,5] varied the cylindrical geometry of premixed gas rocket combustion chambers to determine the effect on high-frequency combustion pressure oscillations. It was found that when the length/diameter ratio was such that the frequencies of the fundamental longitudinal and tangential modes were approximately the same, the amplitudes of oscillation increased significantly. There was an accompanying "shift" of the instability region (on a plane of equivalence ratio vs combustion pressure) to a lower combustion pressure. The changes in the wave shapes of the pressure oscillations observed on an oscilloscope confirmed the belief that an interplay between the two aforementioned modes was occurring. The net effect of mode coupling in a premixed gas rocket was an increase in the severity of combustion instability.

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Systems of First-Order, Nonlinear Differential Equations Convertible to Classical Forms

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IN a recent note, Mason[1] presented classes of first-order systems reducible to Bernoulli and Ricatti type of equations. The material in his note is a particular case of the work reported earlier in Ref. 2, a relevant extract of which is being presented here.

A general first-order nonlinear system can be described by the differential equation

$$\dot{x} + g(x,t) = 0 \tag{1}$$

(dot denotes differentiation with respect to t).

The most general transformation law involving both the dependent and independent variables (unlike the transformation considered by Mason[1] involving the dependent variable only) can be written as

$$X = X(x,t) \tag{2}$$

$$T = T(x,t) \tag{3}$$

Differentiation of Eqs. (2) and (3) with respect to t gives

$$X' = (X_x\dot{x} + X_t)/(T_x\dot{x} + T_t) \tag{4}$$

where the subscripts x and t denote the corresponding partial derivatives and prime denotes differentiation with respect to T.

Substituting Eq. (1) in Eq. (4),

$$X' = (X_xg - X_t)/(T_xg - T_t) \tag{5}$$

For Eq. (5), which represents the given system in the transformed plane, to be amenable to existing methods of analysis, it should conform to one of the classical forms, such as

Bernoulli type:

$$X' = C_1(T)X^n - C_2(T)X \ (n \neq 1) \tag{6a}$$

($n = 0$ represents the linear type)

Ricatti type:

$$X' = C_1(T)X^2 + C_2(T)X + C_3(T) \tag{6b}$$

Separable type:

$$X' = C_1(T)F(X) \tag{6c}$$

Exact equation type:

$$X' = M(X,T)/N(X,T) \tag{6d}$$

$$X' = M(X,T)/N(X,T) \tag{6e}$$

(Here $\partial N/\partial T = \partial M/\partial X$.) Substitution of Eq. (5) into any one of Eqs. (6a–6d) results in a corresponding partial-differential equation. The solutions to such partial-differential equations represent the necessary transformation functions for a given first-order system, Eq. (1), to be reducible to one of the classical forms listed previously.

Although a unique general solution to these partial-differential equations in (X, T) may not be obtainable for any given function $g(x,t)$, it is useful even if a particular solution to any one of these partial-differential equations is obtained.

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Alternatively, conditions on the form of the function $g(x,t)$ for the resultant equation to conform to a known type also can be obtained from the foregoing equations as

$$g(x,t) = \frac{X_t + T_t[C_2(T)X - C_1(T)X^n]}{X_x + T_x[C_2(T)X - C_1(T)X^n]} \quad (7a)$$

$$g(x,t) = \frac{X_t - T_t[C_1(T)X^2 + C_2(T)X + C_3(T)]}{X_x - T_x[C_1(T)X^2 + C_2(T)X + C_3(T)]} \quad (7b)$$

$$g(x,t) = \frac{X_t - T_t[C_1(T)F(X)]}{X_x - T_x[C_1(T)F(X)]} \quad (7c)$$

$$g(x,t) = \frac{X_t - T_t[M(X,T)/N(X,T)]}{X_x - T_x[M(X,T)/N(X,T)]} \quad (7d)$$

Thus any first-order, nonlinear, nonautonomous system given by Eq. (1) along with one of the Eqs. (7a–7d) can be transformed to a classical equation (6a–6d) by the transformation laws (2) and (3).

Both the cases considered by Mason[1] [Eq. (1) and Eq. (7) of Ref. (1)] are but particular examples of Eq. (7a) and (7b) [along with Eq. (1)] with $X_x = f(x)$, $X_t = 0$, $T_t = 1$, and $T_x = 0$. Other cases of interest along with application of the technique to higher-order systems can be found in Refs. 2–5.

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"Catastrophic" Pressure Peaks in Solid-Propellant Combustion

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In two recent Technical Notes,[1,2] comments have been reported concerning the mechanism by which "catastroghic instability" develops in certain solid-propellant combustors. These Notes seek to explain experimental results in an earlier Note.[3] The claim is made that the mechanism arises from coupling of axial and transverse modes under the experimentally observed[3] conditions. It is my opinion that these contributions[1,2] are not original and not clearly relevant to the phenomenon they seek to explain.

As to originality, it should be noted that the earlier paper[3] reporting the phenomenon in question stated that catastrophic instability occurred when the frequencies of unstable transverse modes approached integral multiples of the frequencies of higher unstable axial modes. This statement implied recognition of the modes in question, and the frequency of the relevant transverse modes and axial modes were shown graphically, using the classical acoustic theory on which calculations in Ref. 1 and 2 are based. In this respect, the calculation of an expression relating mode frequencies in Ref. 1 contributes nothing new. Reference 1 goes on to suggest that the critical mode-frequency relationship under discussion may be conducive to some coupling between the modes, but does not propose a mechanism. In the original paper[3] it was noted that no such coupling is possible within the framework of linear analysis defining the modes under consideration, since the modes are orthogonal. One would presumably have to go outside the body of linear analysis to explain the coupling. Since this is not done in Refs. 1 and 2, it is difficult to see how a coupling mechanism can be claimed.

In the second of the recent Notes, the question of mode coupling is explored further, still using linearized analysis. The presence of mean flow or combustion is still neglected; excitation of modes is assumed to occur by a complicated independent vibration of the end surface of the circular cylindrical cavity (as contrasted to a coupled combustion vibration distributed over the side walls in the original experiment). In this analysis, it is shown that the coefficient of a particular term in the series solution becomes relatively large when the driving frequency approaches the mode frequency represented by that term. This observation is a simple demonstration of the classical concept of resonance, and establishes nothing about the severity of oscillations other than the point that the cavity impedance is low at the resonant frequency.† Following the demonstration of resonance in radial modes, the author again calculates the length to radius ratio of a circular cylinder required to give harmonically related frequencies for radial and axial modes, without comment on the purpose of the calculation. The calculations in no way demonstrate coupling between modes, and the boundary conditions used do not simulate those in the original combustion problem.

Aside from the foregoing questions, it is not clear that Refs. 1 and 2 examine the correct problem in the first place. The original paper[3] reported that, for the mode-frequency conditions under discussion, the solid-propellant combustor developed severe pressure peaks that were illustrated by a typical test record. It was stated specifically that oscillations were already present in transverse and axial modes before the pressure peaks developed, and that the peaks occurred when the frequencies of these unstable modes became harmonically related. It was stated that it could not yet be determined whether the oscillatory behavior became still more severe at that time, or whether the pressure peaks were due to an anomalous increase in quasi-steady burning rate under the harmonic frequency condition. This experimental question is still unresolved because of the problems of adequate separation of mode frequencies and accurate amplitude measurement at the high frequencies involved (10–60 k cps). It was noted in the original paper that the burning surface of the propellant became severely rippled during the high amplitude oscillatory behavior, a condition that could be conducive to and symptomatic of large anomalies in burning rate. The recent Notes[1,2] have directed comments to the question of severity of oscillations, a topic on which the original[3] or any other paper presents no information. In any case, it is not clear that one must seek mode coupling to explain the results, since the modes involved were already unstable.

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† No argument about energy transfer into the system is pursued and the author does not seem to recognize that destabilizing combustion oscillations tend to occur at mode frequencies because they are excited by gas oscillations at the frequencies of the most conservative modes.